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Analyzing students' errors and misconceptions in proof construction in abstract algebra course

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Abstract

This study aimed to identify and describe the types of errors and misconceptions made by students in the process of constructing proofs. Using a qualitative approach with a case study design, the subjects were fifth-semester students enrolled in the Abstract Algebra course at Mulawarman University. Three student responses showing significant error patterns regarding the Fundamental Isomorphism Theorem were purposively selected and analyzed based on the Selden & Selden framework. The results indicated that students faced notation, logical, and structural obstacles. Dominant errors included notation inflexibility (E3), reversed logic in surjective proofs due to quantifier neglect (E8), and structural misconceptions in defining kernel and image (M7). Additionally, over-generalization of real number rules in group operations (M5) was observed. These findings suggested that students remained at the procedural-thinking stage and had not yet reached the "Object" stage in their cognitive schema, resulting in a failure to build rigorous deductive arguments. This study recommends instructional interventions emphasizing the visualization of abstract structures and strengthening quantifier logic to minimize students' future epistemological obstacles.

Keywords: Errors, Misconceptions, Abstract algebra, Proof construction, Isomorphism

Abstrak

Penelitian ini bertujuan untuk mengetahui dan mendeskripsikan jenis-jenis kesalahan dan miskonsepsi mahasiswa dalam mengonstruksi bukti. Menggunakan pendekatan kualitatif desain studi kasus, subjek penelitian adalah mahasiswa semester lima mata kuliah Struktur Aljabar di Universitas Mulawarman. Tiga jawaban mahasiswa dipilih secara *purposive* yang menunjukkan pola kesalahan signifikan pada topik teorema fundamental isomorfisma berdasarkan kerangka Selden & Selden. Berdasarkan hasil analisis, diperoleh bahwa mahasiswa mengalami hambatan pada aspek notasi, logika, dan struktural. Kesalahan dominan meliputi infleksibilitas notasi (E3), logika terbalik pada pembuktian surjektif akibat pengabaian kuantor (E8), serta miskonsepsi dalam mendefinisikan kernel dan *image* (M7). Selain itu, terjadi generalisasi berlebihan aturan bilangan real pada operasi grup (M5). Temuan ini menunjukkan mahasiswa masih berada pada tahap berpikir prosedural dan belum mencapai tahap "Objek" dalam skema kognitifnya, sehingga gagal membangun argumen deduktif yang rigor. Penelitian ini merekomendasikan intervensi pengajaran yang menekankan visualisasi struktur abstrak dan penguatan logika kuantor guna meminimalkan hambatan epistemologis mahasiswa di masa depan.

Kata kunci: Kesalahan, Miskonsepsi, Struktur aljabar, Konstruksi bukti, Isomorfisma

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INTRODUCTION

Reasoning and proof are among the five process standards emphasized by the National Council of Teachers of Mathematics (NCTM) in mathematics education (NCTM, 2000). In the mathematical process, proving serves to verify the truth of a proposition while simultaneously explaining the underlying reasoning behind that truth (Herizal; et al., 2024; Rocha, 2019).

At the university level, reasoning and proof lie at the heart of mathematics education, particularly in advanced courses such as Real Analysis, Abstract Algebra, and Linear Algebra. In these subjects, students are not only required to apply formulas but must also construct rigorous formal proofs to verify theorems and explore the underlying logical structures (Chand, 2021; Powers et al., 2010; Stewart & Thomas, 2019). This characteristic reflects the transition from procedural mathematics in secondary school to deductive mathematics in higher education, aiming to develop students' abilities in abstract thinking, critical analysis, and the construction of coherent arguments (Alam & Mohanty, 2024; Nadlifah & Prabawanto, 2017).

One of the courses that demands extensive proof construction is Abstract Algebra. Abstract Algebra encompasses material related to definitions, theorems, propositions, and lemmas within the topics of groups, rings, and fields (Bhattacharya et al., 1994; Lee, 2018). The complexity of this material imposes a high cognitive load, requiring students to not only memorize but also to integrate and apply these formal concepts logically and procedurally across various contexts.

The challenge in mastering Abstract Algebra often arises from its highly abstract nature and the high-level capability required to construct proofs (Agustyaningrum et al., 2023; Subedi, 2020). Many students struggle to understand formal definitions, select and use appropriate theorems, and develop valid proof sequences (Fatmiyati et al., 2020; Weber, 2001). These difficulties frequently result in misconceptions or procedural errors in their solutions (Fatmiyati et al., 2020; Wulan et al., 2021).

Errors in mathematics can be defined as mistakes, wrong steps, or deviations from the correct result. These are usually incidental, such as calculation errors, the use of inappropriate algorithms, or other procedural slips (Parwati & Suharta, 2020). Errors can often be corrected through re-checking or direct feedback. Meanwhile, misconceptions arise from a lack of conceptual understanding or the consistent application of incorrect mathematical principles or rules. A misconception is an understanding that does not align with correct scientific concepts and often persists within the cognitive structure (Hestu Wilujeng et al., 2025; Ridho & Juandi, 2023). Misconceptions can take the form of fundamentally flawed beliefs, such as improper generalization of rules or misclassifying a concept (Fardah & Palupi, 2023; Ridho & Juandi, 2023).

Research on student difficulties in mathematical proof, particularly in group theory, indicates that common errors stem from weak conceptual understanding of fundamental definitions and theorems. For instance, Khafifah, F et al. (2025) identified three main types of errors: conceptual, procedural, and technical, largely caused by a lack of understanding of group properties and imprecision in following proof steps. Similarly, Herizal et al. (2024) identified procedural patterns such as algebraic errors and "proof by example," indicating that students struggle to order logical steps and apply definitions correctly.

Furthermore, students often fail to integrate prior knowledge and select suitable proof techniques, hindering their ability to build complete and consistent proofs (Saha et al., 2024). Other studies confirm that a lack of conceptual understanding and a tendency to rely on rote memorization worsen the ability to select relevant facts and theorems (Panerio & Delideli, 2025).

While research on error analysis and student misconceptions in Abstract Algebra, particularly group theory, is extensive (Elif et al., 2015; Khafifah F et al., 2025; Suradi & Djam'an, 2021; Veith et al., 2022; Yerizon et al., 2019), few studies have conducted an in-depth analysis of errors made during formal proof construction in advanced topics like quotient groups and group homomorphisms. Mastery of quotient groups and homomorphisms is essential for understanding advanced abstract algebra and its applications. These concepts serve as a primary bridge for constructing, comparing, and generalizing complex structures, such as isomorphisms and their fundamental theorems (Cheng, 2023; Mena-Lorca & Parraguez, 2016). Categorizing the extent to which these errors are conceptual (misconceptions) or procedural (errors) remains necessary for optimizing instructional improvements.

The primary objective of this research is to analyze the types of errors and misconceptions students make while constructing proofs. This study contributes specifically by identifying patterns of errors and misconceptions in advanced abstract algebra topics, namely quotient groups and homomorphisms. By identifying "what is wrong" and "why it is wrong" in depth, the findings will provide a robust foundation for designing targeted instructional interventions to minimize the recurrence of these errors in the future.

METHODS

This research employed a qualitative approach with a case study design. The primary objective of this case study was to gain a profound and holistic understanding of the specific types of errors and misconceptions made by students during proof construction. The subjects of this study were fifth-semester students in the

Mathematics Education Study Program at Mulawarman University. The participants were selected through purposive sampling, consisting of three students who exhibited interesting or significant error patterns when solving proof problems related to the Fundamental Isomorphism Theorem, i.e. R1, R2, and R3. The selection was also based on the completeness of their responses, ensuring that the analyzed work covered the entire process, from defining the mapping to the final conclusion. Research data were collected through document analysis of the students' written work. The mathematical problem used to evaluate the subjects' proof construction skills is shown in Figure 1.

> Let K be the group of non-zero real numbers under multiplication and L be the group of positive real numbers under multiplication. The function $\psi: K \rightarrow L$ is defined as $\psi(x) = x^2$. Show that $K/\{1, -1\} \simeq L$.

Figure 1. Test instrument

Data analysis was conducted inductively and interpretatively, utilizing the Selden and Selden Proof Error Taxonomy (Selden & Selden, 1987) as the initial analytical framework. This process involved: (a) step-by-step coding of participants' written proofs to identify each error; (b) classifying these errors into structural and logical categories according to the Selden and Selden framework; and (c) inferring misconceptions by interpreting the reasons or conceptual beliefs underlying these consistent error patterns. Through an in-depth study of these cases, this research aims to uncover how and why specific misconceptions affect students' proof reasoning processes, providing a rich and detailed qualitative description.

RESULTS AND DISCUSSION

The results presented in this section are based on each subject's responses, categorized by the types of errors and misconceptions identified. Each response underwent an in-depth analysis to determine and describe the specific nature of these errors and misconceptions.

Errors and Misconceptions Committed by Subject R1

Based on the responses provided, several errors and misconceptions were identified in Subject R1's work. Consider the following results of Subject R1's proof construction presented in Figure 2. The detailed analysis of the types of errors and misconceptions committed by Subject R1 is presented in Table 1.

The errors committed by Subject R1 in proving isomorphic group structures reflect epistemological obstacles rooted in an inability to transition from a computational-procedural mindset to formal-abstract thinking. Findings regarding notation inflexibility (E3) and the use of information out of context (E10)—where the subject failed to adopt the notation ψ and group K provided in the problem—indicate that students remain at the stage of instrumental understanding (Greeno, 1997). At this stage, students tend to rely on a single example stored in their long-term memory; consequently, when faced with new symbolic variations, cognitive load occurs, hindering flexible thinking (Subedi, 2020). Furthermore, research by Landy et al. (2014) confirms that dependence on standard notation often becomes a barrier for students in understanding the essence of set relations in abstract algebra.

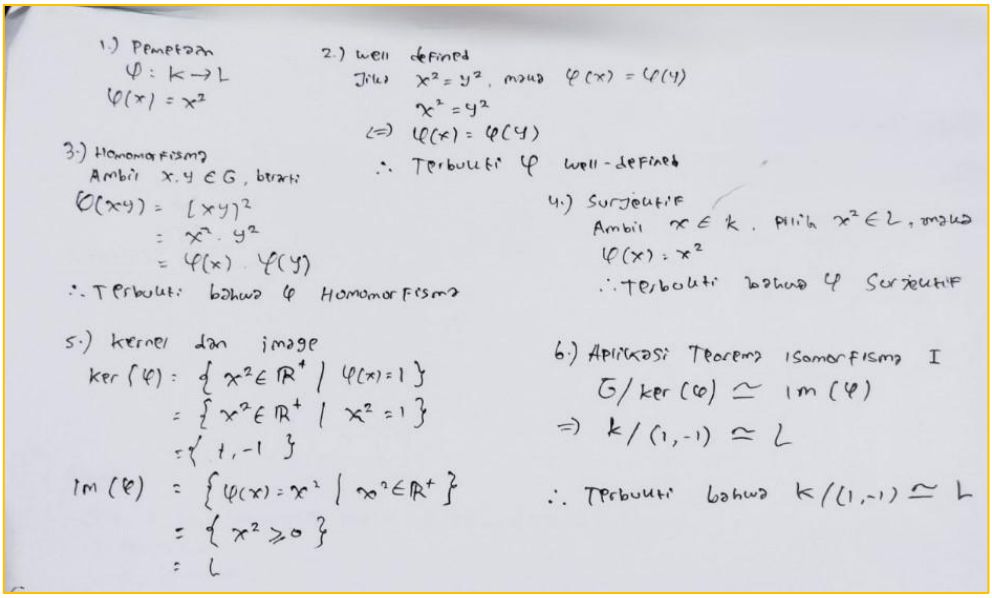

Figure 2. Subject R1's response in constructing the proof

Table 1. Analysis of Error Types and Misconceptions of Subject R1

No	Proof Component	Type of Error and Misconception	Description and Evidence
1	Function Notation	E3: Notation Inflexibility	The problem defines the mapping from K to L using the notation ψ or $\psi: K \to L$. However, Subject R1 consistently used the notation φ or $\varphi: K \to L$, demonstrating a failure to adapt to the specific notation provided in the problem.
2	Homomorphism Domain	E10: Using Information Out of Context	When Subject R1 began showing that the mapping ψ is a homomorphism, the subject wrote "Take $x, y \in G$," whereas the correct domain should be K (the group of non-zero real numbers). The notation G is a general notation for groups taken from other contexts and applied inappropriately here.
3	Surjective Logic	E8: Ignoring and Extending Quantifiers	Subject R1 performed the surjective proof using reversed logic. Ideally, showing surjectivity must start by taking an element from L and demonstrating the existence of an element in K. Subject R1 wrote "Take $x \in K$, choose $x^2 \in L$," which incorrectly suggests that x^2 is selected from L after x is taken from K.
4	Kernel Definition	M7: Interchange of Elements and Sets	This is a fundamental error indicating a structural misconception where Subject R1 defined the kernel as $$\ker(\varphi) = \{x^2 \in \mathbb{R}^+ \mid \varphi(x) = 1\}$$ Since the kernel is a subset of K, the kernel

No	Proof Component	Type of Error and Misconception	Description and Evidence
			elements should be x, not the mapping result x^2 in $\mathbb{R}^+$. Subject R1 confused domain and codomain elements within the set notation.
5	Kernel Calculation	M5: Universal Application of Real Number Rules	Subject R1 demonstrated a logical inconsistency by defining the kernel set as $\{x^2 \in \mathbb{R}^+ \mid x^2 = 1\}$ but providing $\{1, -1\}$ as the final result. This is nonsensical because $-1 \notin \mathbb{R}^+$. This error shows that the subject relied on an intuitive "square root" procedure from real number algebra without maintaining logical consistency within their own set-defined constraints.
6	Image Definition	M7: Interchange of Elements and Sets	Subject R1 defined $Im\ (\varphi) = \{\varphi(x) = x^2 \mid x^2 \in \mathbb{R}^+\}$ The condition $x^2 \in \mathbb{R}^+$ is always true if $x \in K$, whereas the definition should only require $x \in K$. This represents incorrect notation that mixes the requirements of domain and codomain elements.

Moreover, the error in E8 (ignoring and extending quantifiers) regarding the proof of surjectivity, where the subject initiated the proof from the domain instead of the codomain, demonstrates a weak understanding of the order of formal quantifiers (∀ dan ∃). This "reversed logic" phenomenon strengthens the arguments of Mejía-Ramos et al. (2015) dan Weber (2001) that students often view proof as mere symbolic manipulation rather than the construction of a logical argument that must follow the proper mapping direction. This failure is closely linked to fundamental misconceptions in the definitions of Kernel and Image related to misconception M7 (Interchange of Elements and Sets). Subject R1 experienced structural confusion by placing the mapping result x^2 as a member of the kernel, whereas the kernel should be a subset of the domain. From the perspective of APOS theory (Tsafe, 2024), this indicates that the subject has not yet reached the "Object" stage in their cognitive schema, where they can operate functional rules but fail to conceptualize the kernel and image as structurally separate set entities.

Lastly, the subject's tendency to apply real number rules universally (M5) in kernel calculations without referring to the identity element of the multiplicative group indicates an over-generalization of basic arithmetic knowledge. Students often make intuitive leaps by assuming that familiar properties of number systems will automatically apply to newly learned abstract structures. This shows that without strong cognitive connections between the concept of group identity and algebraic procedures, students will continue to struggle in achieving the mathematical rigor required in Abstract Algebra courses (Basir, 2025; Mumu & Tanujaya, 2019). Overall, the misconceptions performed by R1 are not merely technical errors but a representation of mental unreadiness in constructing formal definitions into precise mathematical language.

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Errors and Misconceptions Committed by Subject R2

Based on Subject R2's work in Figure 3, the subject began the proof by writing the mapping, followed by demonstrating that the mapping is well-defined, determining the kernel, establishing that the mapping is a homomorphism, determining surjectivity, and finally drawing a conclusion.

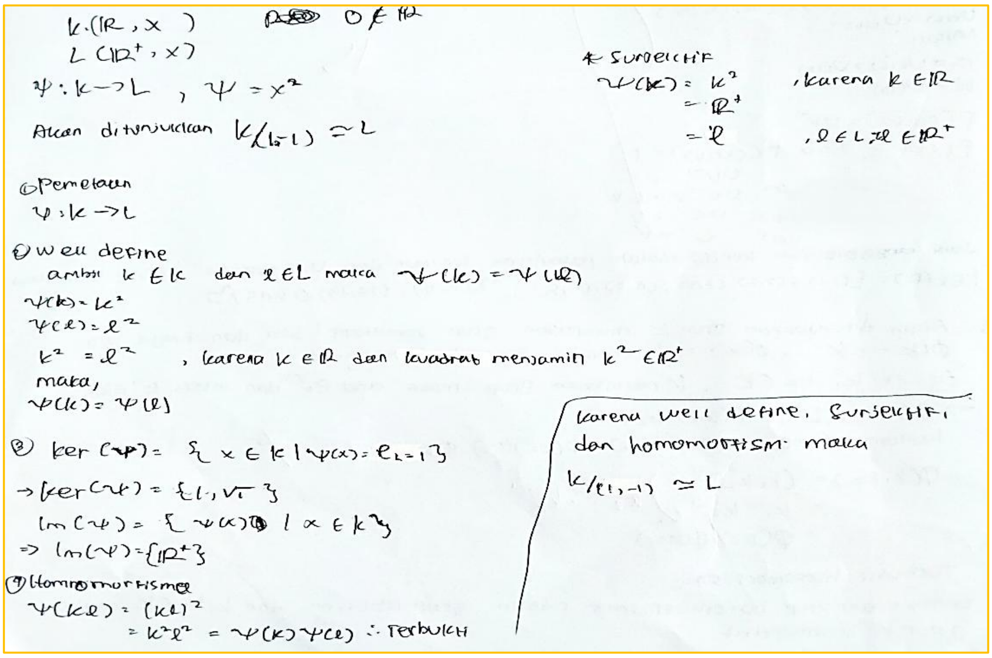

Figure 3. Subject R2's response in constructing the proof

In Subject R2's sequence of work, there is a clear error because the prerequisite for determining the kernel is first showing that the given mapping is a homomorphism. Further details regarding the errors and misconceptions committed by Subject R2 can be seen in Table 2.

Table 2. Analysis of Error Types and Misconceptions of Subject R2

No	Proof Component	Type of Error and Misconception	Description and Evidence
1	Proof Sequence	E5: Disorganized Proof	The sequence of proof steps is illogical because the kernel is discussed before the homomorphism.
2	Well defined	E3: Notation Inflexibility & E1: Overly Broad Symbol	Subject R2 erred in testing well-definedness by taking element k from K and element ℓ from L, then comparing $\psi(k) = \psi(\ell)$, whereas well-definedness only involves domain elements K—specifically, if $k_1 = k_2$, then $\psi(k_1) = \psi(k_2)$.
3	Kernel ($\text{Ker}(\psi)$)	E9: Holes	Although the formal definition of the kernel is correct $\ker(\psi) = \{x \in K \mid \psi(x) = e_L = 1\}$ the subject failed to explicitly solve the set to obtain the result $\ker(\psi) =$

No	Proof Component	Type of Error and Misconception	Description and Evidence
			$\{1, -1\}$.
4	Image ($Im(\psi)$)	E4: Non-Mathematical Context	The subject stated the result $Im\ (\psi) = \{\mathbb{R}^+\}$ atau L without presenting sufficient mathematical arguments to justify that x^2 covers all positive real numbers.
5	Surjective	E8: Ignoring/Extending Quantifiers & E9: Holes	The proof of surjectivity is extremely brief, only claiming $\psi(K) = \mathbb{R}^+ = L$. The subject did not include arguments for the existence of ($x = \sqrt{y}$ or $x = -\sqrt{y}$ for $y \in L$) which is mandatory in a surjective proof.

The analysis of Subject R2's work reveals a pattern of cognitive obstacles focused on the inability to organize a deductive flow and the failure to construct existence arguments. Unlike the previous subject, Subject R2 experienced Disorganized Proof (E5), where proof steps were performed without a logical sequence, such as discussing the kernel before ensuring the mapping structure is a homomorphism. This aligns with findings by Netti et al. (2024), which indicate that failures in constructing mathematical proofs are heavily influenced by fragmented cognitive structures—such as incomplete, disconnected, or immature schemas—causing students and prospective teachers to only link proof steps locally without understanding the proof structure as a coherent whole (Anwar et al., 2023).

In testing the well-defined property, the subject exhibited Notation Inflexibility (E3) and Overly Broad Symbols (E1) by incorrectly comparing elements from the domain K and codomain L. This phenomenon indicates confusion between the definition of a function and set relations, consistent with findings that many students operate function definitions and well-defined properties procedurally without deeply coordinating the domain, codomain, and correspondence rules (Sebsibe et al., 2019; Uscanga et al., 2024; Uscanga & Cook, 2024).

Further errors were identified through the Holes (E9) phenomenon; although the subject could write the formal definition of a kernel, they failed to execute the solution set to completion. This failure suggests that Subject R2 reached a "procedural" level in writing definitions but encountered a deadlock when integrating computational algebraic knowledge into the formal framework. In the surjectivity proof, the subject used Non-Mathematical Text (E4) and Quantifier Neglect (E8) by making brief claims without including the existence argument for the pre-image, $x = \pm\sqrt{y}$. This lack of existential proof confirms Moore (1994) findings that students often rely on visual intuition or oral claims because they perceive formal proofs as too abstract or redundant. Overall, Subject R2 understood "what" needed to be proven definitively but failed to explain "how" the argument is constructed rigorously and logically within abstract algebra standards.

Errors and Misconceptions Committed by Subject R3

Figure 4 shows the problem-solving process carried out by Subject R3. From this figure, we can observe how the proof strategy chosen by the subject triggers several types of errors and misconceptions. A detailed analysis of the types of errors and misconceptions experienced by Subject R3 can be found in Table 3. This analysis highlights how mistakes in proof strategy and symbolic manipulation lead to the construction of an invalid proof.

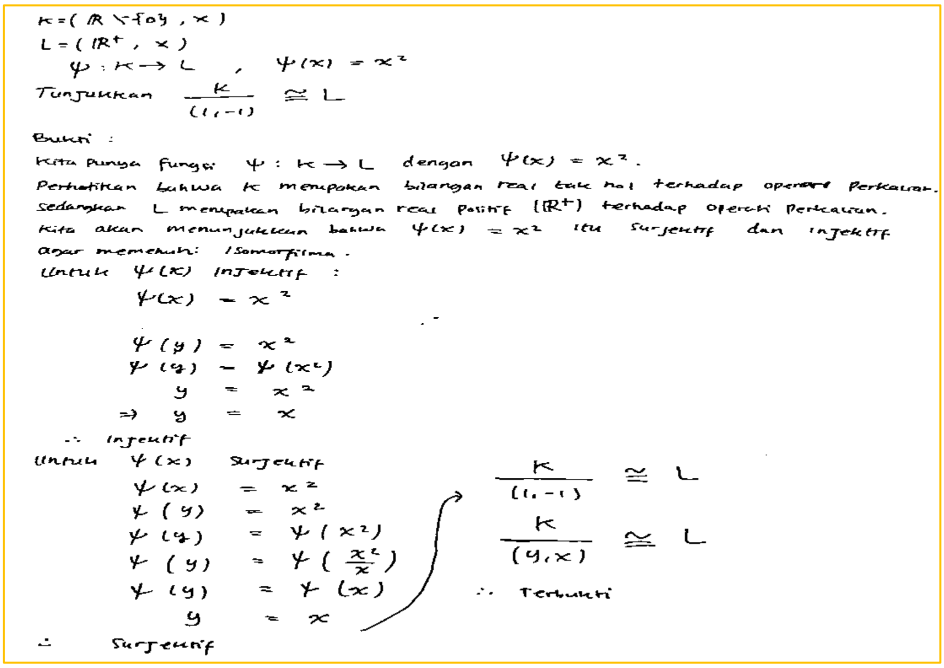

Figure 4. Subject R3's response in constructing the proof

Table 3. Analysis of Error Types and Misconceptions of Subject R3

No	Proof Component	Type of Error and Misconception	Description and Evidence
1	Initial Strategy	E6: Invalid/Missing Proof & E5: Disorganized Proof	The subject did not prove the Homomorphism property at all. Instead, the subject attempted to prove $K/\{1,-1\} \simeq L$ directly (which was incorrect) rather than utilizing the First Isomorphism Theorem.
2	Injective Property	M4: Inconsistent Application of Rules & M3: Reversal of Definition/Theorem	The subject failed to recognize that $x^2 = y^2$ in $K = \mathbb{R}^*$ berarti $x = \pm y$. He claims that $y = x$ is fundamentally wrong, proving that the mapping is actually not injective.
3	Surjective Property	E6: Invalid/Missing Proof & E8: Ignoring Quantifiers	The mathematical manipulation in the surjectivity proof was nonsensical, beginning with $\psi(y) = \psi(x)$ and continuing in a manner that did not align with the definition of surjectivity, which requires finding a pre-image $x \in K$ for every $y \in L$.

No	Proof Component	Type of Error and Misconception	Description and Evidence
4	Conclusion	E1: Overly Broad Symbol & E4: Non-Mathematical Text	The use of quotient group notation was incorrect and ambiguous, such as $K/(y,x)$ and $(1,-1)$ (using ordered pairs instead of the set $\{1,-1\}$).

In-depth, Subject R3's failure in the isomorphism proof is rooted in the inability to use formal definitions as tools for proving. This aligns with findings that many students view definitions as memorized text rather than operational tools for building structured deductive arguments (Schneider, 2020). Such results can be categorized as errors E6 (invalid/missing proof) and E5 (disorganized proof), reflecting a failure to build a systematic proof scheme, consistent with studies by Norton et al. (2025) on student difficulties in constructing introductory proofs.

In the injectivity section, the subject's error in concluding $x^2 = y^2 \Rightarrow x = y$ within group $\mathbb{R}^*$ indicates an epistemological obstacle in the form of overgeneralization from high school algebra rules, where students are accustomed to single-solution scenarios (Schneider, 2020). Specifically, the subject failed to recognize that in the group of non-zero real numbers, the equation actually has two solutions: $x = \pm y$. This error occurs because the subject remains at the Action level of understanding in APOS theory, capable only of performing routine procedural manipulations without structural reflection on identity elements and group operation properties (Arnon et al., 2014). At this level, group definitions and properties have not been encapsulated as objects that can be flexibly manipulated, making such errors very common.

Furthermore, the errors made regarding surjectivity can be classified as E6 (invalid proof) and E8 (ignoring quantifiers). The subject performed illogical symbolic manipulations, such as writing $\psi(x) = \psi(y)$ without a systematic procedure to find a pre-image x for every y in the codomain. This pattern indicates a weak understanding of quantified statements and the functional relationship between domain and codomain elements. Studies on the transition from natural to formal language show that many students struggle to consistently interpret function symbols, dependent variables, and the quantifiers "for every" or "there exists"; consequently, reasoning about surjectivity often devolves into notation manipulation devoid of logical meaning (Kwon & Park, 2025).

Finally, in the conclusion, Subject R3 committed errors related to E1 (overly broad symbols) and E4 (non-mathematical text). The subject used ordered pair notation $K/(1,-1)$ or $K/(y,x)$ to represent the quotient group instead of writing the correct set of cosets, such as $K/\{1,1\}$.. This demonstrates a cognitive inability to encapsulate the coset concept into a single abstract object. Rather than seeing a coset as a "single point" in the quotient structure, the subject reverted to more familiar schemas like Cartesian products and ordered pairs. Studies on epistemological obstacles in advanced mathematics suggest that when students lack adequate visualization and representation schemas for abstract objects (such as quotient groups or partitions), they tend to replace them with more concrete but ontologically incorrect structures (Schneider, 2020).

CONCLUSION

Based on the results and discussion above, it can be concluded that students experience significant cognitive obstacles in integrating their understanding of structural concepts with formal proof capabilities. This is characterized by the emergence of fundamental misconceptions, such as the confusion between domain and codomain elements in defining the Kernel (M7), as well as patterns of disorganized proof (E5) and logical "holes" (E9) that lead to the failure of constructing valid arguments regarding surjectivity and homomorphisms. The reliance on real number algebraic intuition, which triggers over-generalization (M4/M5), along with routine procedural manipulation behaviors, confirms that student understanding remains stalled at the "Action" level of APOS theory. At this level, they fail to perform structural reflection on the operational boundaries of groups. This condition culminates in ontological misconceptions when representing quotient groups using ambiguous symbols (E1), demonstrating a failure in the process of encapsulating the coset concept into a single object. By identifying in depth "what" and "why" these errors occur, this study successfully maps specific failure patterns in quotient group and homomorphism materials. These findings serve as a crucial foundation for designing instructional interventions based on cognitive transitions to minimize similar obstacles in the future.

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