Systematic Review of Spatial Thinking in Mathematics Education

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ABSTRACT

The objective of this study is to perform a comprehensive analysis of existing research on spatial thinking abilities using a Systematic Literature Review (SLR). To achieve this, five search engines (Scopus, SpringerLink, ScienceDirect, ProQuest, and ERIC) were utilized during various stages, including identification, screening, eligibility, inclusion, and data analysis. Adhering to the PRISMA reporting rules, a total of 22 articles were included, selected mainly from the sorted studies. The results of our research indicate that there were four studies conducted using a mixed method approach, while nine studies utilized a quantitative approach, and another nine focused on qualitative methods. The number of studies investigating spatial thinking has consistently increased over the years. The findings section revealed that the majority of studies on this subject were authored by researchers from the USA, followed by authors from Australia, Turkey, Israel, Germany, Spain, China, Slovakia, Japan, Mexico, South Africa, and the United Kingdom.

Keyword: mathematics education, spatial thinking, spatial ability, systematics review

INTRODUCTION

Spatial thinking refers to the cognitive process by which the brain interprets the positions and shapes of objects. In the field of mathematics, it encompasses the study of numbers, diagrams, understanding quantities, structural patterns, changes, and spatial relationships. Proficiency in spatial cognition, particularly in geometry, including polygons, surface area, plans, and elevations, is crucial for students to excel in mathematics education. Various terminologies are used in the literature to characterize and explain spatial ability. Recent research has shown that spatial thinking is flexible and can be enhanced through training, real-world experiences, and educational interventions (Uttal & Cohen, 2012). Moreover, spatial thinking finds practical applications in mathematics education, involving physical, visual, and imaginative interactions with objects (Korkmaz & Birol Tekin, 2019). Scholars emphasize the importance of intrinsic-dynamic thinking (also known as visuospatial processing) in accurately translating 2-dimensional (2D) into 3-dimensional (3D) shapes, sizes, orientations, locations, or directions of objects (Sorby, 2009; Taylor & Hutton, 2013).

Spatial thinking involves mentally constructing and manipulating both two- and three-dimensional objects, as well as perceiving an item from different perspectives (Fiantika et al., 2018). Previous research provides evidence that students with strong spatial thinking abilities learn better through visualizations compared to those with weaker spatial thinking skills

(Newcombe, 2022). Moreover, individuals with higher spatial thinking abilities tend to perform better in Science, Technology, Engineering, and Mathematics (STEM) fields, particularly in mathematical discovery, learning, and communication (Katie & Gilligan-Lee, 2022). These differences in spatial thinking play a role in predicting interest and success in STEM disciplines. Certain studies suggest that teaching mathematics with an emphasis on spatial thinking is highly effective, especially in the early stages of mathematical education. Spatial thinking supports the development of numerical skills, including understanding volume, numerical classification, numbering, place value, and arithmetic theories and skills. Consequently, encouraging spatial thinking from an early age is essential for a person to gain a deeper understanding of mathematical concepts. Without spatial skills, the cognitive abilities necessary for creative and inventive thought are limited.

Numerous recent studies have focused on spatial thinking. Recent correlational research suggests a connection between spatial thinking and mathematics in young children, although this link is not fully recognized in primary school students. Two long-term studies have found that early spatial skills, particularly mental rotation and visuospatial memory, are associated with later spatial and mathematical abilities in young children (Gunderson et al., 2012). Spatial thinking is a multi-dimensional concept that encompasses various talents, involving the mental visualization of spatial features and understanding spatial concepts. Interestingly, while individuals with higher spatial thinking performed better on the change-detection task, there was no evidence to support the idea that they were better at utilizing symmetry compared to individuals with lower spatial thinking abilities. To address these gaps and gain more insights, a systematic literature review was conducted to analyze articles related to spatial thinking in mathematics education. The research review includes five research questions that will provide further information in this area.

1. What research methodology was utilized in the study?
2. How are the spatial thinking studies distributed across different publication years?
3. In which countries are spatial thinking studies predominantly conducted?

LITERATURE REVIEW

Before delving into the current trends and interest in teaching spatial thinking, it is crucial to comprehend spatial thinking in the context of human development and learning. Spatial thinking involves skills utilized by individuals to organize, manipulate, and imagine objects or concepts. The extent to which spatial thinking is innate remains a significant question influencing its application in education. Scholars have attempted to develop theories regarding the development of spatial thinking (Newcombe & Learmonth, 2005). Spatial thinking is a form of complex thinking that integrates concepts related to space, representation, and reasoning.

This perspective enables the identification of spatial issues, the formulation of suitable solutions, and the explanation of those solutions (National Research Council, 2006). Spatial talents can be distinguished through three basic ways: spatial orientation, which involves perceiving the positions of objects in space; mental rotation, which is the ability to mentally visualize the rotation of objects; and spatial visualization, which pertains to perceiving complex spatial patterns (Mix & Cheng, 2012).

Mental rotation, one of the spatial abilities that has received considerable attention in mathematical research, has been found to significantly aid individuals in performing missing term addition and subtraction tasks. Moreover, it is theorized that mental rotation plays a crucial role in supporting geometric reasoning by providing the mental models on which such reasoning relies (Kovacevic, 2017). A novel typology of spatial talents, based on theoretical considerations, was developed by examining the literature on spatial training (Uttal et al., 2013). This typology distinguishes between dynamic and static dimensions, as well as intrinsic and extrinsic dimensions. Intrinsic dimensions refer to an object's parts and their relationships to each other within the object itself. On the other hand, extrinsic dimensions pertain to how an object relates to a larger set of objects.

The investigation of mathematical talents has a rich historical background, as discussed by Mix et al. (2016), dating back to the early distinction between conceptual and procedural aspects of various mathematical disciplines like number concepts, algebra, measurement, fractions, ratios, and geometry. These essential tasks can be further divided into specific categories: Campbell (2004) distinguished between numerical skills, such as number comparison and representation, and mathematical problem-solving skills, involving abstract representations of mathematical relations derived from contextual problems. Gaber and Schlimm (2015) classified numerical and geometric skills as fundamental mathematical abilities. Additionally, spatial thinking encompasses advanced disciplinary spatial reasoning based on expert knowledge and reasoning within different academic fields like structural geology, surgery, chemistry, and mathematics. However, although spatial reasoning plays a role in tasks requiring physical prediction, it does not fully account for all the individual differences observed in intuitive physics.

METHOD

To investigate the concept of spatial thinking in mathematics education for a Systematic Literature Review (SLR), an extensive literature review was undertaken. To ensure transparent reporting of the study's purpose, methods, and findings, the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) statement, introduced by Moher et al. in 2009, was utilized. The relevant journal articles were summarized using PRISMA guidelines for the SLR. The SLR process was divided into three main stages: planning, conducting, and reporting the review. These phases guided our approach and data collection procedures, supporting the execution of this SLR.

To begin with, the primary focus of this SLR is to determine an appropriate title. Given the limited coverage of the topic, "Spatial Thinking in Mathematics Education" was chosen as the subject, aiming to uncover new insights in this area. From this title, key search terms were deduced, including spatial thinking, spatial ability, spatial skills, mathematics education, and mathematics. These keywords were combined using Boolean operators "AND" and "OR" to construct a search string, resulting in more precise and fruitful outcomes. The methodology for conducting the literature search is presented in Table 1.

Table 1. Literature search technique and strategy

Keyword	The Strings and combinations of keywords
Spatial Thinking	("Spatial Thinking" OR "Spatial Ability" OR "Spatial Skills") AND
Mathematics Education	("mathematics education" OR "mathematics")

The subsequent phase involved implementing the PRISMA guidelines to conduct an extensive search for relevant publications. This step facilitated data collection by accessing existing research data from published articles. The PRISMA selection process consisted of four key steps: identification, screening, eligibility, and inclusion. Next, acceptable articles for the study were identified by conducting searches using the keywords listed in Table 1: (("Spatial Thinking" OR "Spatial Ability" OR "Spatial Skills") AND ("mathematics education" OR "mathematics")). Five search engines, namely Scopus, SpringerLink, ScienceDirect, ProQuest, and ERIC, were employed to conduct the searches. The results from each search engine were numbered using the provided keywords. Specifically, 624 results were retrieved from Scopus, 945 from SpringerLink, 2,398 from ScienceDirect, 10,721 from ProQuest, and 1,379 from ERIC databases. In total, the latest search produced 16,067 results that required filtering. The flow of the PRISMA protocol is depicted in Figure 1.

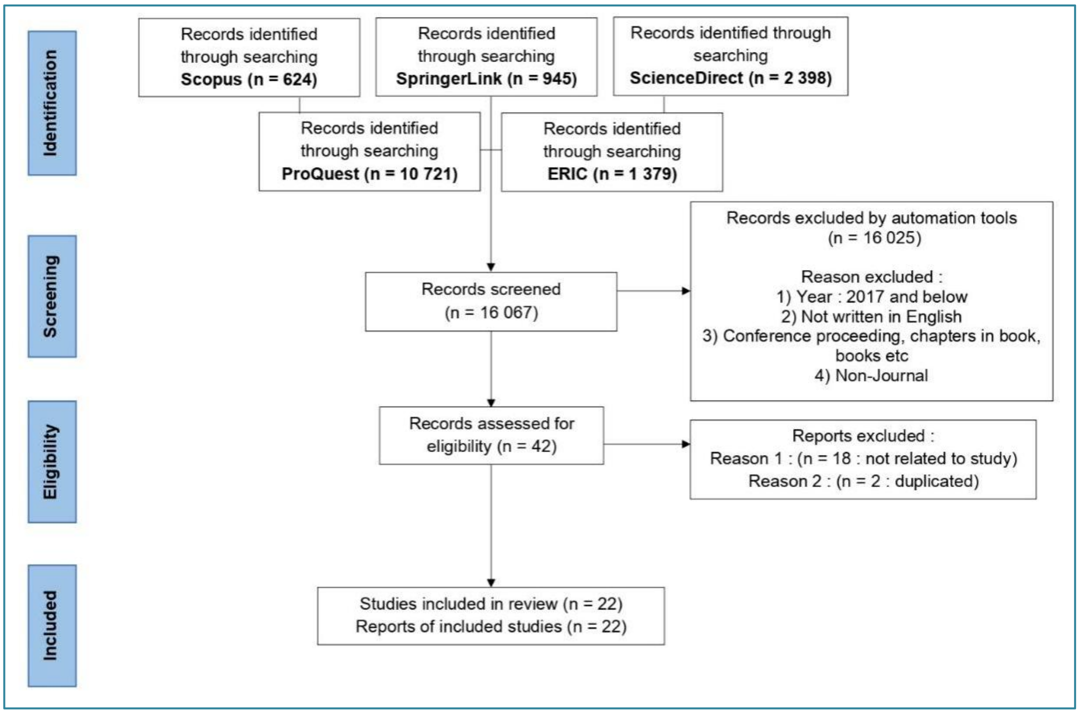

Figure 1. PRISMA protocol flow chart

The screening process involves identifying papers from the literature survey that meet the criteria for inclusion in the review. Out of the initial 16,025 results, automation tools were used to sort the articles based on factors such as document accessibility, language used, document type, and publication year. The selected article titles should be directly related to the theme of spatial thinking in mathematics, specifically focusing on spatial reasoning skills concerning 2D representations of 3D geometrical shapes in grades 4 to 9 (Taro Fujita et al., 2020). To ensure the inclusion of relevant and recent findings, articles published between 2018 and 2022 were considered. For inclusion in the review, only original research published in scholarly publications was considered, while books, newspapers, conference papers, systematic literature reviews, and other types of publications were excluded. Additionally, articles had to be written in English as their primary language, and access to the full text had to be freely available. The inclusion and exclusion criteria are outlined in Table 2, and the sorting procedure was applied to each search engine.

The eligibility criteria for this study encompass a combination of research question features and specific types of studies that have addressed those questions. It refers to meeting the necessary requirements to be considered for inclusion. After the automation tool sorted the initial 16,025 documents, 42 articles remained for further analysis and exclusion, as they met the criteria. During this phase, all authors collaborated to assess the eligibility of each article based on its abstract. This collaborative approach ensured efficient evaluation while ensuring that all requirements for the review were properly identified. Various aspects, such as relevant titles, appropriate topics, and alignment with the target sample, were assessed during this phase.

Following the evaluation, 22 publications were deemed qualified for inclusion in the systematic literature review, as they fulfilled all the requirements. On the other hand, 18 articles were deemed unsuitable for the topic requirements, either because they focused on subjects other than mathematics or were not centered on spatial thinking. Additionally, two articles were removed as duplicates. Consequently, 22 articles were selected for review and analysis to address the research question and conduct the systematic literature review, based on the inclusion criteria outlined in Table 2.

Table 2. Criteria of inclusion and exclusion

Criteria	Inclusion	Exclusion
Article title and content	Relevant title and met the study's requirements	Irrelevant title and did not meet the study's requirements
Year of publication	From 2018 until 2022	Publications other than the specified range
Type of publication	Original studies and journal article type only	Publications other than the specified range (Non-Journal)
Language	English	Others
Field of article study	Mathematics Education	Others
Sample Size	Primary school students, secondary school students, teachers and university students	Out of the specified sample size
Accessibility	Full-text articles	Preview articles and required a payment

The quality of the included articles and the review methodology are influenced by the study's validity and reliability. To enhance data validity and reliability and minimize the risk of bias, a rigorous outline following the PRISMA protocol and adherence to the inclusion criteria were employed as strategies. The review of publications aimed to understand how spatial ability had been defined in previous research, its benefits and limitations in mathematics education, and to aid the authors in analyzing the research question for reporting the findings.

In this systematic literature review, the collected data from various articles that were part of the search were carefully evaluated and presented. Matrices were employed to extract important data from each publication, which was then used in this review. The reporting section of the review included the following components: prior studies, a literature review abstract, an introduction, a research question, a conceptual framework, methodology, findings, a discussion, and a conclusion, along with a list of references. The abstract provided a concise overview of the subject, highlighting the themes and foundations of the review. The introduction aimed to underscore the significance of the subject and its relevance to the chosen previous research under study. To ensure comprehensiveness, five research questions were formulated to assess the data and provide answers based on the findings. A theoretical framework was developed to elaborate on the review and address related issues identified. The conclusions of this review were drawn by synthesizing the findings to address each research question comprehensively.

After reviewing the relevant literature, the findings were carefully reported and documented. The objective was to address all the research questions in this review comprehensively. To ensure comprehensive coverage and coherence, the review's findings were presented using tables, figures, and explanations. This approach aimed to avoid any omission of important research topics and enhance the overall flow of the review. The primary findings were summarized and discussed after their presentation. As there were five sections in the findings corresponding to the five research questions, emphasis was placed on highlighting the most significant findings relevant to the objectives of the study. Furthermore, the study's strengths and limitations were discussed to provide insights for future research improvements. To gain a holistic perspective on the study's effectiveness, comparisons were made with prior research. The limitations encountered during the study were also elucidated to provide clarity and guidance for future research endeavors. Reporting procedures were established to ensure the information is accessible and usable for long-term learning. The review concluded by offering suggestions for further research and explaining the relevance of the findings to future academic endeavors.

Thematic analysis was the chosen method for analyzing the studies in this systematic literature review (SLR). Thematic analysis involves identifying patterns or themes in qualitative data. Braun and Clarke (2006) recommend it as the first qualitative method to learn, as it provides valuable skills applicable to various analyses. To complete the literature review, significant findings and information from earlier studies were collected and utilized, primarily to address the research question. The SLR's thematic analysis consisted of three steps. First, the text in the documents was coded line by line, resulting in a total of 22 papers after sorting the included studies to gather the data. The analysis process also considered the country of publication, as diverse perspectives were deemed essential. A connected theory was provided to explain how the theory relates to the subject of mathematical education. Additionally, the review included the study designs to highlight the various types of research that could be conducted concerning spatial thinking. In the third stage, the development of descriptive themes closely aligned with the studies included in the first step took place. Moreover, the final analytical themes were established, allowing the authors to go beyond the investigations' findings and create fresh interpretations, explanations, or hypotheses. These pieces of data were integrated into the abstract, introduction, methods, findings, and discussion of the research. The referenced studies served to describe and explain the implementation of these findings into the review's context. The gathered findings were classified as qualitative data.

RESULT AND DISCUSSION

Result

The findings in this domain were classified based on their research strategies, publication years, and the countries where they were conducted. The research question addressed whether spatial thinking contributes to enhancing knowledge and proficiency in mathematics. The assigned research question focused on the total number of studies on spatial thinking published from 2018 to 2022. The subsequent sections provide answers to the research question that was assigned.

The initial research topic centered around the research design employed in studies related to Spatial Thinking. Figure 2 presents the distribution of studies based on their research approaches. Four studies were conducted using a mixed method approach, as indicated by Jessica et al. (2020), Linda Medina Herrera et al. (2019), Dorit Patkin et al. (2019), and Rule et al. (2019). Additionally, there were nine studies that utilized a quantitative approach, including works by González et al. (2022), Silverman et al. (2022), Harris et al. (2020), Caroline Cohrssen et al. (2019), Garofalo et al. (2019), Velázquez et al. (2021), Lee et al. (2019), Hubber et al. (2019), and Mulligan et al. (2017). Likewise, nine studies were focused on qualitative methods, including research by Nagar et al. (2022), Kus et al. (2022), Zhang (2021), Patahuddin et al. (2020), Bongani et al. (2019), Hubber a et al. (2019), Baranovat et al. (2018), Reinhold et al. (2020), Gotz et al. (2022), and Fujita et al. (2020).

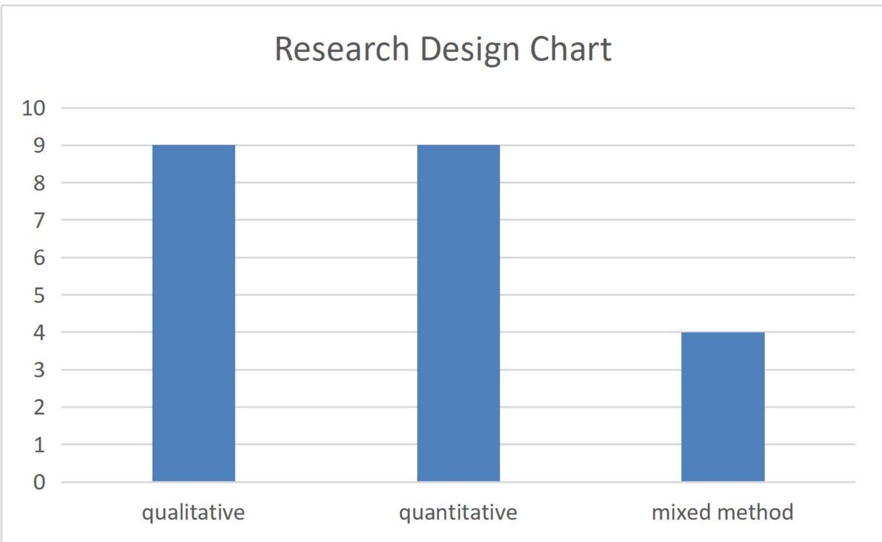

Figure 2. Research design

The second research question addressed the relationship between the number of studies conducted on spatial thinking and the publication year of those studies. According to the data presented in figure 3, the research on spatial thinking in mathematics education has consistently increased over the years. A total of 22 papers were published on this subject between 2017 and 2022. In 2017 and 2018, one article was published each year. However, in 2019, the number of publications significantly increased to seven, indicating a rising interest in spatial thinking research. This upward trend continued until 2020, with a slight decline of five articles. In 2021, there was a further decrease of two articles, but in 2022, the number of publications bounced back with an additional five articles compared to the previous year. The peak in publications occurred in 2019, surpassing the number of articles published in any previous year. The consistent growth in research indicates the increasing significance of spatial thinking in mathematics education and reflects the growing interest among students and researchers in this field.

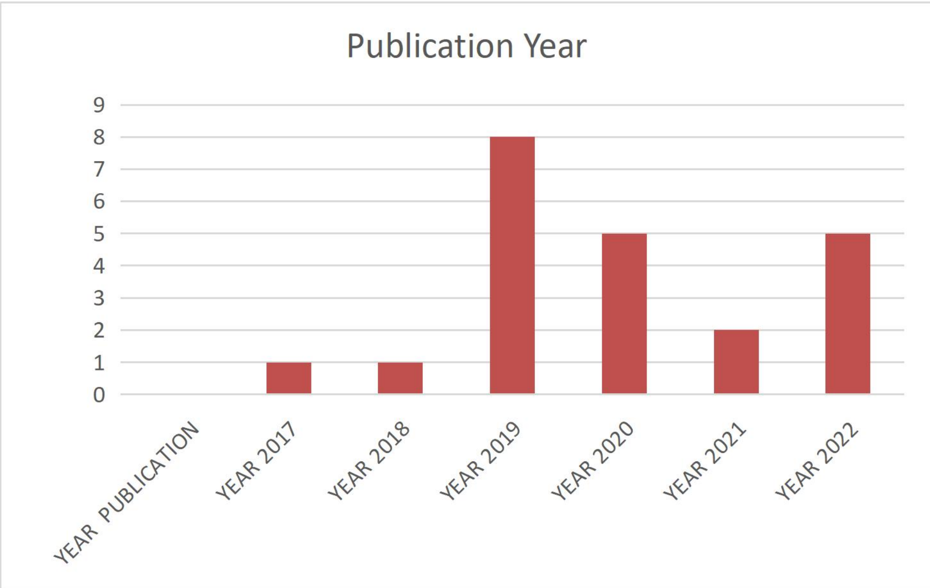

Figure 3. Publication year

The third research aspect focused on the distribution of studies on spatial thinking across different countries. As depicted in figure 4, the United States of America led with the highest number of publications in the subject of spatial thinking, specifically five articles, accounting for 22.73% of the total coverage among the 22 articles. Australia ranked second with four publications, making up 18.18% of the total coverage. Turkey, Israel, and Germany each contributed three publications between 2018 and 2022, collectively covering 9.09% of the subject matter. On the other hand, Spain, China, Slovakia, Japan, Mexico, South Africa, and the United Kingdom each had only one study published on spatial thinking in the context of mathematics education. The future research on spatial thinking in mathematics education may play a small but significant role in the development of crucial cognitive abilities. This could potentially lead to improved performance in more advanced mathematics courses, which often emphasize spatial concepts worldwide.

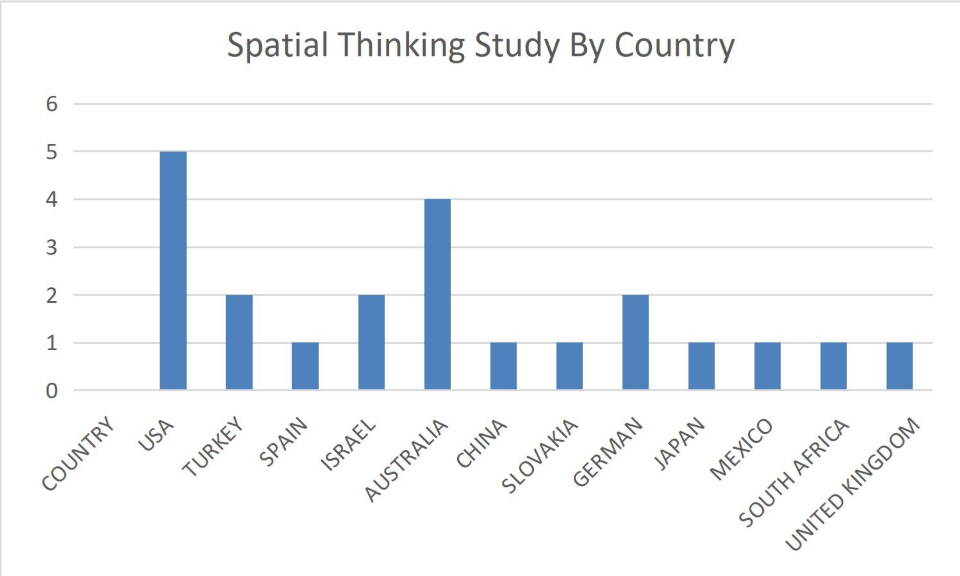

Figure 4. Spatial thinking study by country

Discussion

We found that nine articles utilized a quantitative research design as the main method in their studies. Quantitative approaches focus on precise measurements and use statistical, mathematical, or numerical analysis of data collected through surveys, polls, or other forms of data collection, including computational methods applied to historical statistical data. The primary objective of quantitative research is to gather numerical data and generalize it across populations or explain a specific phenomenon. Similarly, nine articles employed a qualitative research design. Qualitative research involves studying the nature of phenomena, making it particularly suitable for exploring the reasons behind observations or their absence, evaluating complex multi-component interventions, and focusing on intervention improvements. The findings indicate that many researchers, both in quantitative and qualitative studies, opted for this method to achieve a large sample size and gather accurate data related to spatial thinking in mathematics education. Additionally, a few researchers used a mixed-method research design, combining both qualitative and quantitative research approaches in four articles. This approach aims to gain a comprehensive understanding of the concept of spatial thinking.

The following research question examines the distribution of studies on spatial thinking in mathematics education based on their publication year. It was observed that by the year 2019, there was a notable increase in the number of research topics conducted worldwide. This study also analyzed the distribution of spatial thinking studies in mathematics education according to their publication year, as specified during the screening process, which covered the interval from 2017 to 2022. Among the 22 articles reviewed, a consistent growth in related studies was observed in 2017 and 2018, with one article published each year, even though spatial thinking in mathematics education is still considered a relatively new research topic. The research articles covered the years 2017 to 2022, with only two articles published in both 2020 and 2022. This suggests that the research topic is still considered novel and may explain the lower number of publications in those years. However, in 2021, only two articles were found, indicating a potential decline in interest or publication on the topic during that year.

Regarding the distribution of the study by country, the findings section revealed that the USA had the highest number of prominent authors contributing to studies on spatial thinking in mathematics education, followed by Australia, Turkey, Israel, Germany, Spain, China, Slovakia, Japan, Mexico, South Africa, and the United Kingdom. This information allowed us to identify the various countries that have contributed to research related to spatial thinking in mathematics education. Notably, the United States of America (USA) had the most significant presence, with five articles published on the subject. This suggests a high level of interest and focus on spatial thinking in mathematics education within the USA. Conversely, seven countries, including Spain, China, Slovakia, Japan, Mexico, South Africa, and the United Kingdom, each published only one article. This indicates a potential lack of awareness or limited research attention towards the contribution of spatial thinking in mathematics education in these countries. In light of these findings, it is essential for all countries to take action and conduct more studies on spatial thinking in mathematics education to benefit future generations and enhance their understanding and application of this important concept.

CONCLUSION

This comprehensive analysis of existing research on spatial thinking abilities through a Systematic Literature Review (SLR) provided valuable insights into the current state of knowledge in this field. The review involved thorough searches using multiple search engines, rigorous screening based on PRISMA guidelines, and the inclusion of 22 relevant articles. The findings highlight the growing interest in spatial thinking as reflected in the increasing number of studies conducted over the years. The implications of this study are multi-faceted. Firstly, the identification of the USA as the leading contributor to research on spatial thinking in mathematics education suggests the significance of this topic in the American educational landscape. Other countries, especially those with limited representation in the literature, can take cues from the USA and allocate resources to further explore the impact of spatial thinking on mathematics learning and achievement. The distribution of studies across different research designs (mixed methods, quantitative, and qualitative) indicates the need for a varied and holistic approach when investigating spatial thinking. Researchers can draw from diverse methodologies to gain a comprehensive understanding of how spatial thinking influences mathematical cognition and problem-solving.

Furthermore, the steady increase in the number of studies on spatial thinking reflects the ongoing interest and recognition of its relevance in education. This trend encourages further exploration and validates the importance of spatial thinking as a crucial cognitive skill for mathematics learning and beyond. In terms of practical implications, educators and policymakers should take note of the significance of spatial thinking in mathematics education. Integrating spatial thinking development into the curriculum, pedagogical strategies, and educational interventions may enhance students' mathematical understanding and problem-solving abilities. Overall, this study's findings underscore the importance of spatial thinking as an essential cognitive skill in mathematics education and highlight the need for continued research and application of spatial thinking concepts in educational practices.

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